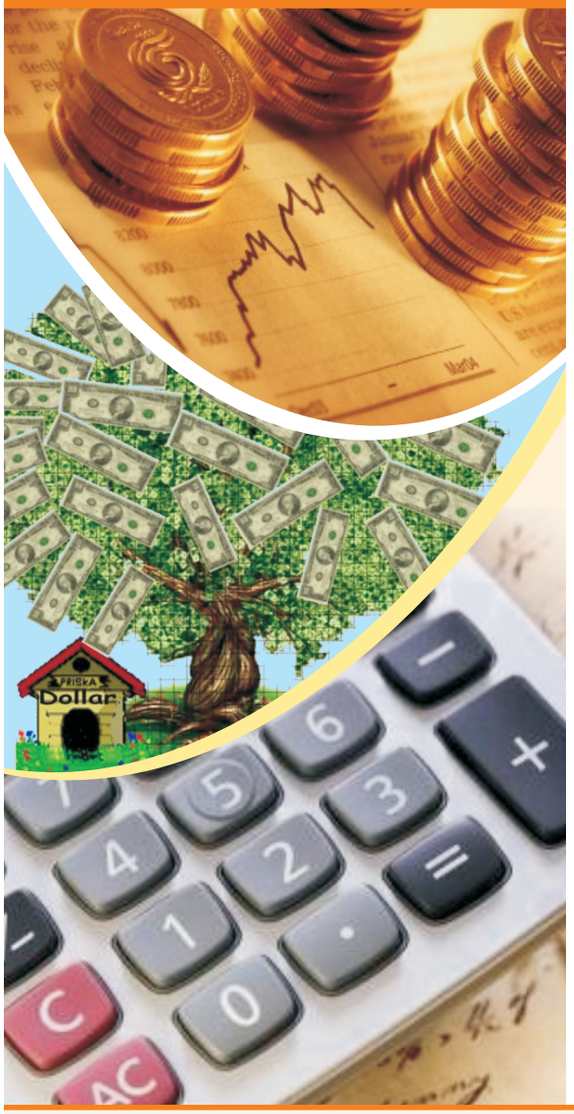
UNIT 1
CHAPTER 1.1

Integers



"Money does not grow on trees." This is one among a few lines that we often hear when our income remains fixed but our expenses soar. Without proper guidance, people keep on purchasing things and may sometimes get into debt. To avoid such situations, they seek the help of financial advisors to plan the use of their money effectively and efficiently. Likewise, big companies too need a team of financial advisors to manage their funds and ensure that the company's financial position remains sound.

Financial wizards such as Subhash Lakhotia and Robert G. Allen are synonymous with financial success. They advise people in making wise investments to earn profits or save tax on their incomes. Financial advisors study the short- and long-term goals of people and guide them in managing their wealth. Firms such as Merrill Lynch and JP Morgan Chase & Co. provide similar financial services to their clients. Therefore, to go into the field of finance, knowing about numbers in general and integers in particular would be an added advantage.

Robert G. Allen
Investment Advisor

Subhash Lakhotia
Tax Expert

Financial Advisors

- Advise individuals and companies to make wise decisions regarding their finances and investments
- Manage their client's finances for fulfilment of short- and long-term goals
- Advise their clients on plans to fund higher education of their children and to make retirement plans
- Work towards minimising the risk of loss and maximising the wealth of their clients

What you already know...

- Natural numbers are counting numbers that start from 1 and go on infinitely as 2, 3, 4, 5, ……..,98,...and so on.
- Whole numbers are same as natural numbers with the addition of zero. So, whole numbers are 0, 1, 2, 3, 4,……. These numbers too go on infinitely.
- Integers include all whole numbers along with their negatives. Thus, the integers are …….–4, –3, –2, –1, 0, 1, 2, 3 , 4,….. Since whole numbers are infinite, it follows that integers too are infinite.
- Integers can be added, subtracted, multiplied and divided with each other.
- The absolute value of an integer is its distance from zero. It is basically the positive value of an integer. Therefore, the absolute value of 3 is 3 and the absolute value of –3 is also 3 because each number is at a distance of 3 units from 0.

Do you remember what a number line of integers is and how it appears? Plot the numbers 4, –10, 0, 8, –2, –1, 3, –11 and 15 on a number line.

What you need to know...

While playing games, you need to follow certain rules. Similarly, certain rules are to be followed while carrying out operations on integers. Let us learn about these rules so that you can play around with integers.

Addition of Integers

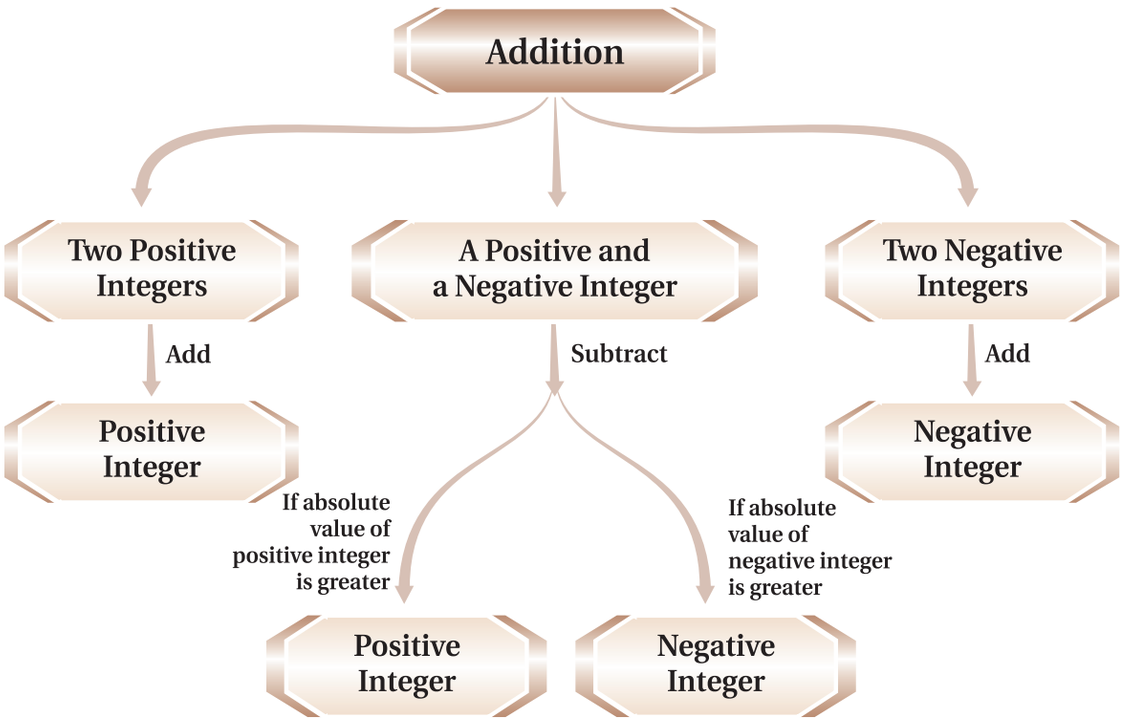
- Two positive integers, when added, give a positive integer.

  Example: 5 + 6 = 11
- Two negative integers, when added, give a negative integer. To add two negative integers, ignore the signs, add the numbers and then allot the negative sign to the answer.

  Example: (–5) + (–6) = –5 – 6 = –11
- To add a positive integer and a negative integer, subtract the two numbers. Next, allot a positive sign to the answer if the absolute value of the positive integer is greater and a negative sign to the answer if the absolute value of the negative integer is greater.

  Example: (–5) + 6 = 1 and 5 + (–6) = –1

The flow chart given below shows the rules for addition of integers:



Subtraction of Integers

- If we subtract two positive integers, we get a positive integer if the first integer is greater and a negative integer if the second integer is greater.

  Say, if we want to subtract 5 from 6, we write it as

  $6 - 5 = 1$

  On the other hand, if we want to subtract 6 from 5, we write it as

  $5 - 6 = -1$

- If we subtract a positive integer from a negative integer, we get a negative integer.

  Say, if we want to subtract 6 from –5, we write it as

  $(-5) - (+6)$

  $= -5 - 6$

  $= -11$

- If we subtract a negative integer from a positive integer, we get a positive integer.

  For example, to subtract –5 from 6, we write it as $6 - (-5)$. So, we get:

  $6 - (-5)$

  $= 6 + 5$

  $= 11$

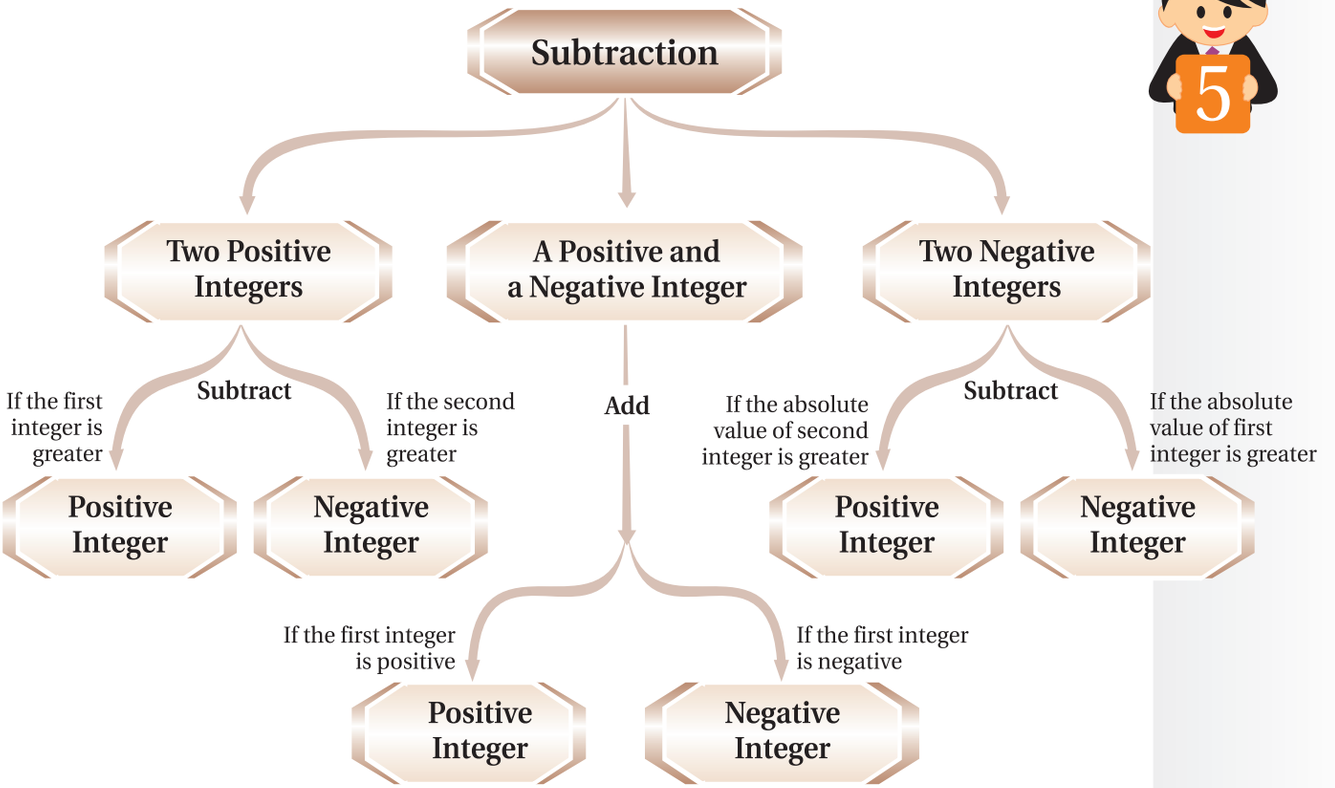
- If we subtract two negative integers, we get a positive integer if the absolute value of second integer is greater and a negative integer if the absolute value of first integer is greater. For example:

(a) $(-5)-(-6)$ $\quad$ (b) $(-6)-(-5)$

$=(-5)+6$ $\quad$ $=(-6)+5$

$=1$ $\quad$ $=-1$

The flow chart given below shows the rules for subtraction of integers:



Addition and Subtraction of 2-digit and 3-digit integers

Now that the rules are clear to you, do you think you can add and subtract 2-digit and 3-digit integers? Let us have a look!

We add and subtract 2-digit and 3-digit integers by applying the same rules as learnt above. Here are a few solved examples.

Addition:

Two positives

30 + 20 = 50

A positive and a negative

(a) 42 + (–32) = 42 – 32 =10

(b) 23 + (–45) = 23 – 45 = –22

Two negatives

(–85) + (–40) = –85 – 40 = –125

Subtraction:

Two positives

(a) 320 – (+40) = 320 – 40 = 280

(b) 25 – (+438) = 25 – 438 = –413

A positive and a negative

(a) 239 – (–42) = 239 + 42 = 281

(b) (–109) – (+42) = –109 – 42 = –151

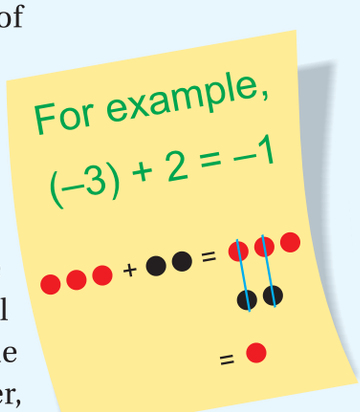
Two negatives

(a) (–330) – (–204) = –330 + 204 = –126

(b) (–28) – (–314) = –28 + 314 = 286

What you need to do...

Let us play an interesting game to see how well you know addition of integers. Work in groups of three. Select symbols of your choice to represent positive and negative integers. For example, you can select black dots to represent positive integers, red dots to represent negative integers and a white dot to represent zero. Write mathematical equations for addition of two positive integers, two negative integers, and a positive and a negative integer. Now, illustrate them on a pastel sheet using the selected symbols, as shown. You will also need to give your poster a name and include a key at the top of the poster, like, black dots = positive integer, red dots = negative integer, white dot = zero. Do not use the same equation more than once. Remember, the best poster will be awarded.



Warm-up session – I...

1. Use the rules of addition and subtraction of integers to solve the following sums:
   (a) $22 + (-11)$
   (b) $415 - 81$
   (c) $7 - (-12)$
   (d) $188 - 98$
   (e) $3 - (-11)$
   (f) $73 + (-21)$
   (g) $99 - (-161)$
   (h) $64 - (-64)$
   (i) $72 - 123$
   (j) $(-190) - (-219)$

2. Make a flow chart to show the rules for addition of integers.
3. State the rules used for subtraction of integers.
4. Subtract the following:
   (a) –315 from 450
   (b) 0 from –70
   (c) –31 from 31
   (d) –234 from 405
   (e) –315 from –213
5. Frame mathematical expressions for each rule of subtraction of integers and solve them.
6. One integer is greater than the other by 7. If one number is –22, find the other number.
7. Match the following:

(a)	–10 + 4	15
(b)	–2 – 3	–11
(c)	12 + 3	– 6
(d)	–15 + 4	– 5

What you need to know...

You will be amazed to know how rapidly the field of financial mathematics has grown over the past decade and how opportunities have emerged in careers in these sectors. Have you heard of a financial wizard? Well, he/she is a person who has the ability to deal with numbers with immense ease. So, let us consolidate our knowledge and review the operations of multiplication and division of integers.

Multiplication and Division of Integers

The rules that apply to multiplication of integers are the same that apply to division of integers. Here is how we go about multiplying or dividing two integers. Ignoring the signs, multiply or divide the numbers and then allot appropriate signs to the answers. The signs are allotted as per the following **three golden rules**:

- Two positive integers when multiplied or divided **always** give a positive integer.

  Example: $34 \times 8 = 272$ and $364 \div 4 = 91$

- Two negative integers when multiplied or divided **always** give a positive integer.

  Example: $(-20) \times (-5) = 100$ and $(-20) \div (-5) = 4$

- A positive integer and a negative integer when multiplied or divided **always** give a negative integer. This is irrespective of whether the greater number is positive or negative.

  Example: $20 \times (-5) = -100$ and $20 \div (-5) = -4$

  $(-20) \times 5 = -100$ and $(-20) \div 5 = -4$

Using these golden rules, you can multiply or divide any two integers. Let us have a look at some more examples.

Multiplication

Two positive integers	A positive and a negative integer	Two negative integers
553×17	$40 \times (-30)$	$(-420) \times (-30)$
$= 9{,}401$	$= -1{,}200$	$= 12{,}600$

Division

Two positive integers	A positive and a negative integer	Two negative integers
$200 \div 25$	$(-660) \div 66$	$(-729) \div (-27)$
$= \frac{200}{25}$	$= \frac{-660}{66}$	$= \frac{-729}{-27}$
$= 8$	$= -10$	$= 27$

What you need to do...

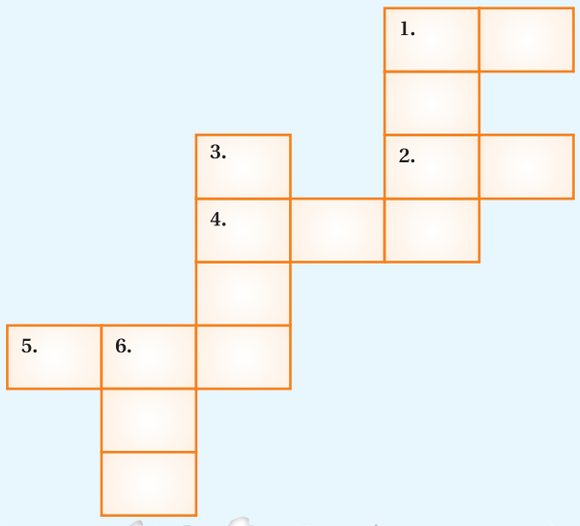
Let us solve a crossword. You can solve the puzzle by finding answers to the mathematical expressions given below. Note that the negative sign of an answer will occupy a separate box while positive integers need to be entered without writing the positive sign.

Across

1. $420 \div (-70)$
2. $(-7) \times (-11)$
4. $(-50) \times (-4)$
5. $(-60) \times (-6)$

Down

1. $10 \times (-17)$
3. $(-6) \times 40$
6. $(-20) \times (-30)$



Warm-up session – II...

1. State the rules for multiplication and division of integers.
2. Solve the following sums:
   (a) $(-20) \times (123)$
   (b) 12×65
   (c) $(-45) \times (-21)$
   (d) $(-123) \div (-3)$
   (e) $540 \div (-12)$

What you need to know...

For managing a client's money in a safe and wise manner, financial advisors need to know their short- and long-term plans. These plans may include anything from setting up a business, investing in stocks and shares, or planning for a steady income after retirement. Likewise, to be able to appreciate the beauty of integers, you first need to know more about them. Let us look at a few properties of integers.

Properties of Integers

You already know the properties of numbers, i.e. closure property, commutative property, associative property and distributive property. Now, let us see whether these properties hold for addition, subtraction, multiplication and division of integers.

Properties of integers with respect to addition

1. Closure property

Add any two integers and you will always get an integer. In other words, we say that integers are **closed** with respect to addition.

Example: $(-4) + (-7) = -11$, which is an integer

and $4 + (-7) = -3$, which is an integer

2. Commutative property

Two integers added in any order will always give the same integer. So, we say that integers are **commutative** with respect to addition.

Example: $(-4) + (-7) = (-7) + (-4)$

$$-4 - 7 = -7 - 4$$

$$-11 = -11$$

So, $\mathbf{(-4) + (-7) = (-7) + (-4)}$

3. Associative property

While adding three integers, no matter how we group them, we will always get the same integer. So, we say that integers are **associative** with respect to addition.

Example: $8 + \{(-4) + 3\} = \{8 + (-4)\} + 3$

$$8 + (-1) = 4 + 3$$

$$7 = 7$$

So, $\mathbf{8 + \{(-4) + 3\} = \{8 + (-4)\} + 3}$

4. Distributive property

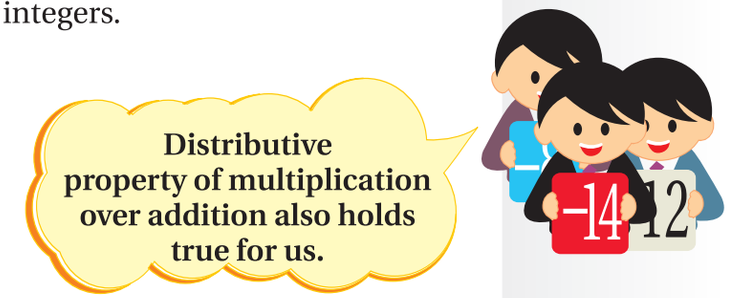
The product of an integer and the sum of two integers can be broken down as the sum of two products. So, we say that the **distributive** property of multiplication over addition holds true for integers.

Example: $(-8) \times (4 + 3) = (-8 \times 4) + (-8 \times 3)$

and $-8 \times 7 = -32 + (-24)$

$$-56 = -56$$

So, $\mathbf{(-8) \times (4 + 3) = (-8 \times 4) + (-8 \times 3)}$



Properties of integers with respect to subtraction

1. Closure property

Subtract any two integers and you will always get an integer. So, we say that integers are **closed** with respect to subtraction.

Example: $(-4)-(-7)=3$, which is an integer

and $4-7=-3$, which is an integer

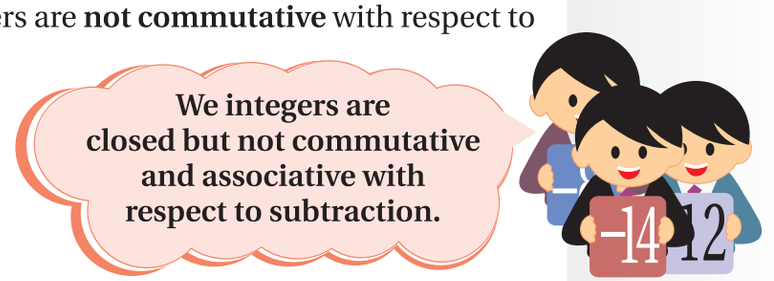
2. Commutative property

The order in which two integers are subtracted determines the answer that you get. Therefore, we say that integers are **not commutative** with respect to subtraction.

Example: $(-4)-(-7)=3$

and $(-7)-(-4)=-3$

So, $\mathbf{(-4)-(-7)\neq(-7)-(-4)}$



3. Associative property

While subtracting three integers, the order in which we group them determines the answer we get. Therefore, we say that integers are **not associative** with respect to subtraction.

Example: $8-(-4-3)=8-(-7)=15$

and $\{8-(-4)\}-3=12-3=9$

So, $\mathbf{8-(-4-3)\neq\{8-(-4)\}-3}$

4. Distributive property

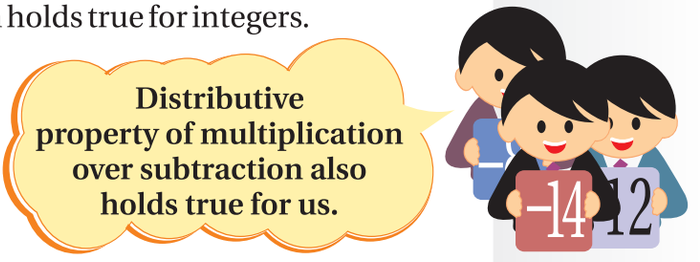
The product of an integer and the difference of two integers can be broken down as the difference of two products. Therefore, we say that the **distributive** property of multiplication over subtraction holds true for integers.

Example: $-8\times(4-3)=(-8\times4)-(-8\times3)$

$(-8)\times1=(-32)-(-24)$

$-8=-8$

So, $\mathbf{-8\times(4-3)=(-8\times4)-(-8\times3)}$



Properties of integers with respect to multiplication

1. Closure property

Multiplication of any two integers always gives an integer. Therefore, we say that integers are **closed** with respect to multiplication.

Example: $-4\times(-7)=28$, which is an integer

and $4\times(-7)=-28$, which is an integer

2. Commutative property

Two integers, multiplied in any order, always give the same integer. Thus, we say that integers are **commutative** with respect to multiplication.

Example: $(-4) \times (-7) = (-7) \times (-4)$

$28 = 28$

So, $\mathbf{(-4) \times (-7) = (-7) \times (-4)}$

3. Associative property

While multiplying three integers, no matter how we group them, we always get the same answer. Thus, we say that integers are **associative** with respect to multiplication.

Example: $8 \times (-4 \times 3) = \{8 \times (-4)\} \times 3$

$8 \times (-12) = (-32) \times 3$

$-96 = -96$

So, $\mathbf{8 \times (-4 \times 3) = \{8 \times (-4)\} \times 3}$

We integers are closed, commutative and associative with respect to multiplication.

Properties of integers with respect to division

1. Closure property

Dividing any two integers will not always give an integer; thus, we say that integers are **not closed** with respect to division.

Example: $2 \div 4 = \frac{1}{2}$ and $\frac{1}{2}$ is not an integer.

2. Commutative property

The order in which two integers are divided determines the answer that we get. Hence, we say that integers are **not commutative** with respect to division.

Example: $-4 \div (-2) = 2$, which is an integer

while $(-2) \div (-4) = \frac{1}{2}$, which is not an integer

So, $\mathbf{-4 \div (-2) \neq (-2) \div (-4)}$

Closure, commutative and associative properties with respect to division does not hold true for us.

3. Associative property

While dividing three integers, the order in which we group them determines the answer we get. Therefore, we say that integers are **not associative** with respect to division.

Example: $(8 \div 4) \div (-2) = 2 \div (-2) = -1$

and $8 \div \{4 \div (-2)\} = 8 \div (-2) = -4$

So, $\mathbf{(8 \div 4) \div (-2) \neq 8 \div \{4 \div (-2)\}}$

*Did you notice that while **all** the given properties hold true for multiplication of integers, **not even one** of the three properties hold true for division of integers?*

What you need to do...

Get into teams of eight. Each team further divides itself into two groups (of four each), group A and group B. Members of group A cut out four square pieces of cardboard and write the four operations on them, one operation per cardboard piece. Each member of group A then picks up one piece. Members of group B do the same, except that instead of writing the four operations, they write the four properties, one property per cardboard piece. Having done this, the team of eight invites another team of eight, who too have followed the same procedure. A member from the invited team chooses two members from the other team, one from group A and the other from group B. He/She then has to tell, with the help of an example, whether the particular property held by the person from group B holds for the particular operation held by the person from group A in integers. Each member of each team must get a turn. The answers, along with the examples, are to be recorded.

Group A

+

−

×

÷

Group B

Associative

Commutative

Distributive

Closure

Warm-up session – III...

1. Name the property with the operation that holds for the following mathematical equations:

   (a) $(-8) + (-7) = -15$, which is an integer

   (b) $20 \times (-2) = (-2) \times 20$

   (c) $18 + \{(-4) + 23\} = \{18 + (-4)\} + 23$

What you need to know...

Financial experts such as Subhash Lakhotia and Robert G. Allen apply their expertise in numbers to find solutions to various financial problems of their clients. Likewise, let us apply our knowledge of properties of integers to solve complex problems.

Simplification using the Properties of Integers

You can use the properties of integers to solve complex problems by making your calculations easier, as shown below:

Example 1: Solve 25×98.

Solution: Rather than simply multiplying the two numbers, you could write 98 as $(100 - 2)$. We try to write any one number (98 in this case) as the sum or difference of two numbers that are easier to multiply. Let us see how this method makes solving the problem easier.

25×98

$= 25 \times (100 - 2)$

$= (25 \times 100) - (25 \times 2)$

$= 2{,}500 - 50 = 2{,}450$

That was easy!

Example 2: Solve $46 \times 73 + 27 \times 46$.

Solution: We can see that both the products have a common number, i.e. 46.

By using the distributive property of multiplication over addition, we can write it as:

$46 \times (73 + 27) = 46 \times 100 = 4{,}600$

Example 3: Solve $45 + 88 + 55 + 12$.

Solution: Use the associative property of addition to group the numbers in such a manner that it becomes easier to add them.

So, $45 + 88 + 55 + 12$

$= (45 + 55) + (88 + 12) = 100 + 100 = 200$

Example 4: Solve $25 \times 67 \times 4$.

Solution: Use the associative property of multiplication to group the numbers in such a manner that it is easier to multiply them.

So, $25 \times 67 \times 4$

$= (25 \times 4) \times 67$

$= 100 \times 67 = 6{,}700$

Example 5: Solve $57 \times 63 - 57 \times 13$.

Solution: We can see that both the products have a common number, i.e. 57.

By using the distributive property of multiplication over subtraction, we can write it as:

$57 \times (63 - 13) = 57 \times 50$

$= 2{,}850$

What you need to do...

Get into groups of four and formulate a numerical expression each. The expressions should be such that they can be solved using the associative or distributive properties of integers. A member from the first group comes forward and writes the four expressions of his/her group on the board. Everyone solves the problems and then checks the solutions within their group. Each group will give a combined solution. Each group gets as many points as the number of correct solutions. Similarly, the member from the second group gives four numerical expressions and this activity continues till every group has had a chance to write their numerical expressions on the board. Keep a record of the points scored, and the team with the maximum points is the winner.

Warm-up session – IV...

1. Solve the following sums by using properties of integers:

(a) $25 \times 36 \times 4$

(b) $12 \times 78 + 12 \times 22$

(c) $49 \times 5 \times 20$

(d) $34 \times 5 \times 120$

(e) $45 \times 44 + 50 \times 45 + 45 \times 6$

(f) $75 \times 20 \times 15$

(g) $(-25) \times (-30) - (-25) \times 42$

(h) $(-15) \times 102$

(i) $15 \times (-37) + 85 \times (-37)$

(j) $25 \times 80 \times 32$

What you know now...

- ☐ Two positive integers, when added, give a positive integer; two negative integers, when added, give a negative integer.
- ☐ To add a positive integer and a negative integer, subtract the two numbers. Next, allot a positive sign to the answer if the absolute value of the positive integer is greater and a negative sign to the answer if the absolute value of the negative integer is greater.
- ☐ If we subtract two positive integers, we get a positive integer if the first integer is greater and a negative integer if the second integer is greater.
- ☐ If we subtract a positive integer from a negative integer, we get a negative integer and if we subtract a negative integer from a positive integer, we get a positive integer.
- ☐ If we subtract two negative integers, we get a positive integer if the absolute value of the second integer is greater and a negative integer if the absolute value of first integer is greater.
- ☐ Two positive or negative integers, when multiplied or divided, always give a positive integer; a positive integer and a negative integer, when multiplied or divided, always give a negative integer.
- ☐ Integers are closed, commutative and associative with respect to addition and multiplication. They are closed but not commutative and associative with respect to subtraction.
- ☐ Distributive property of multiplication over addition and subtraction holds true for integers.
- ☐ Closure, commutative and associative properties of integers do not hold true with respect to division.

Time to chat...

http://blog.themillenniumschools.com

Welcome to educhat

Web screen for 'Integers Crazy'

Angad: Hey friends, have you ever thought that while playing ludo, we use integers to calculate our score?

Khushi: Yes, that's right. I can even express the depth of a submarine moving 600 feet below sea level as –600 feet.

Urooj: Hi guys! Can I too join this interesting discussion?

Khushi: Most welcome. You know the properties of numbers also apply to integers.

Angad: Oh yeah, integers are closed with respect to addition, subtraction and multiplication but not with respect to division.

Urooj: The best part I learnt was the rules that are to be applied while performing operations on integers very carefully.

Khushi: Oh really! So, tell me if you have 72 red marbles and 45 black marbles and Angad needs 125 marbles in all to play a game, will you be able to meet his demand?

Urooj: Oh that's so simple; I will fall short by 8 marbles.

Angad: Wow, Urooj you have beaten a calculator too.

Urooj: Thanks. I solved this using the three golden rules for multiplication and division of integers.

Angad: I know positive × positive or negative × negative will give a positive integer but positive × negative will give a negative integer and the same holds true for division too.

Khushi: Hey, you can even use the properties of integers for rearranging numbers to compute faster.

B I U Arial 15 pt

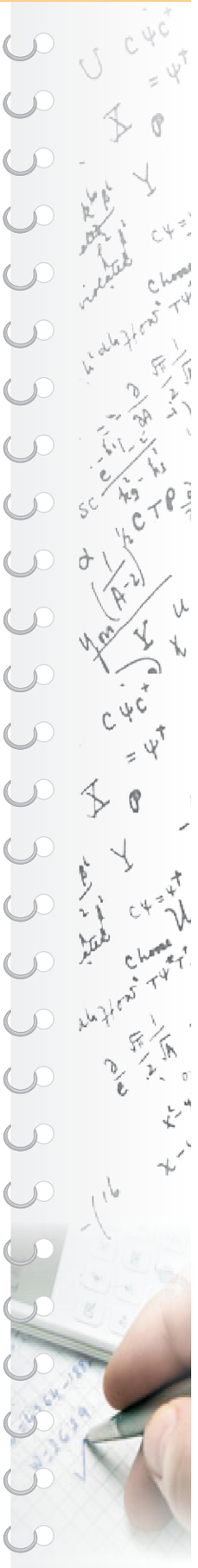
Ok guys, I will take your leave for now. Catch you later. Let us meet again at http://blog.themillenniumschools.com

Send

Done

Time to work out...

1. State the difference between whole numbers and integers.
2. Fill in the blanks:
   (a) Two negative integers, when added, give a _____ integer.
   (b) Two negative integers, when multiplied, give a _____ integer.
   (c) Two negative integers, when divided, give a _____ integer.
   (d) A positive and a negative integer, when multiplied or divided, give a _____ integer.
3. Write the following integers in descending order:
   –78, 94, 0, –56, 43, 12, –4, –87, 34, –34
4. State whether the statements given below are true or false. Correct the false statements.
   (a) Integers are not closed with respect to multiplication.
   (b) The commutative property of integers holds true with respect to addition.
   (c) The associative property of integers holds true with respect to division.
   (d) The commutative property of integers holds true with respect to division.
   (e) Integers are not closed with respect to division.
5. Carry out the following additions and subtractions:
   (a) $-84-45=$ _____ (b) $32-80=$ _____
   (c) $34-(-6)=$ _____ (d) $-50-(-46)=$ _____
   (e) $-90+110=$ _____ (f) $-76+26=$ _____
6. Carry out the following multiplications and divisions:
   (a) $56\div(-7)=$ _____ (b) $(-121)\div 11=$ _____
   (c) $(-144)\div(-12)=$ _____ (d) $25\times(-4)=$ _____
   (e) $(-14)\times 3=$ _____ (f) $(-12)\times(-6)=$ _____
7. Use the given property of integers to fill in the blanks:
   (a) $56+45=$ _____ + _____ (commutativity)
   (b) $75\times 37=$ _____ × _____ (commutativity)
   (c) $67+(45+32)=$ (_____ + _____) + _____ (associativity)
   (d) $(84\times 30)\times 67=$ _____ × (_____ × _____) (associativity)
   (e) $28\times(10+2)=$ _____ + _____ (distributivity)
   (f) $73\times(100-1)=$ _____ – _____ (distributivity)



8. Solve the following using the properties of integers:

   (a) 67×99 (b) $32 + 78 + 68$

   (c) $25 \times 98 \times 4$ (d) $23 \times 45 + 45 \times 77$

   (e) $66 + 99 \times 66$ (f) 48×4

9. In an examination, three marks are given for every correct answer and two negative marks for every incorrect answer. If Ravi gets 20 correct and 40 incorrect answers, what is his score?

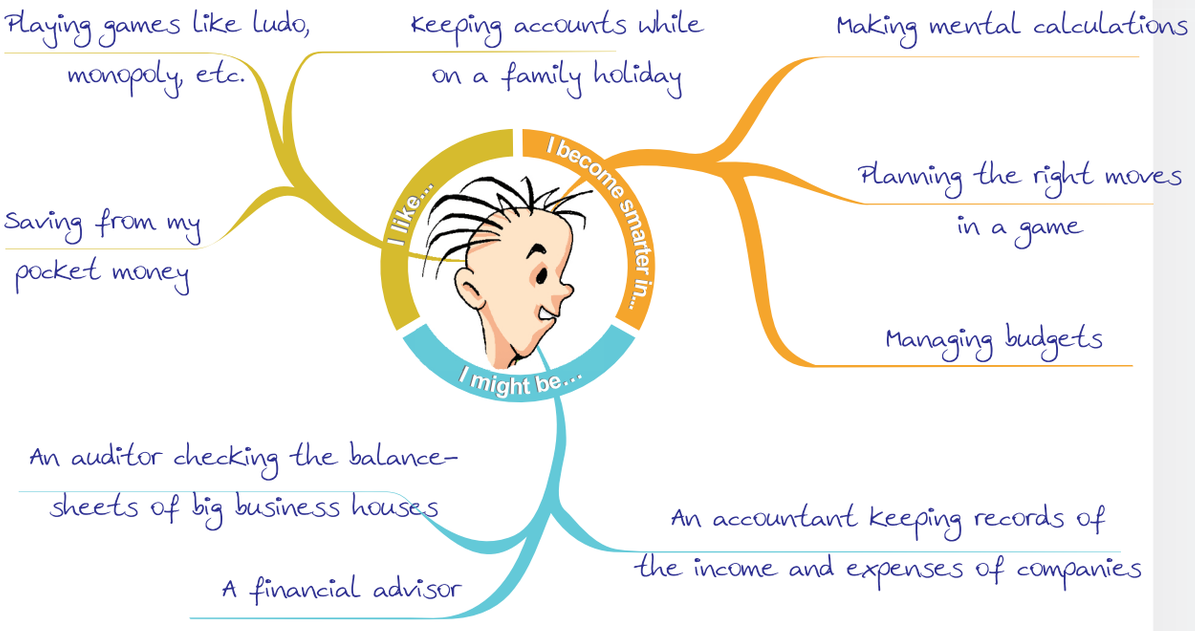
10. In an entrance exam, two marks are given for every correct answer and one negative mark for every incorrect answer. If Meena gets 48 correct answers and her total score is 84, how many questions did she answer incorrectly?

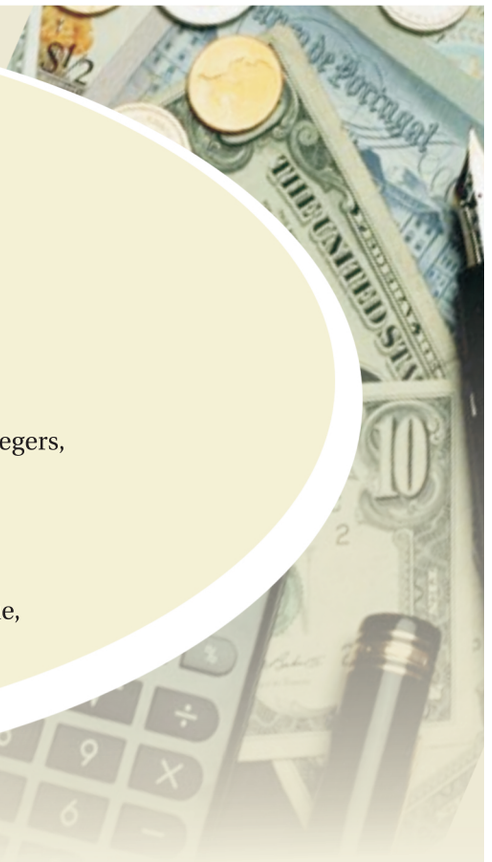
11. In a class, the math teacher announces that he would give two negative marks for bad behaviour and five positive marks for good behaviour. Rajan is caught four times for bad behaviour and gets marks for good behaviour on two occasions. What are Rajan's final marks?

For the apprentice...

As an apprentice to a financial advisor, you have been asked to analyse the profits of a company. You need to collect some data from the Internet. You are expected to make a series of calculations using the properties of integers to arrive at your result.

What you can aspire to be...





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For more help, access... www.mathguru.com

Tool Kit... Cardboard, white paper, coloured sheets, scissors, glue, felt pens, pastel sheets